AES 2009 Part 2 Exam July 4 2018

1. a) $V = \frac{Q}{\pi R^2} = \frac{1.67 \cdot 10^{-8}}{\pi (0.0008)^2} = 0.00831$ m/s

$Re = \frac{DV\rho}{\mu} = \frac{(0.0016)(0.00831)(1000)}{(0.001)} = 13.3$

laminar. What is $Re\,Pr\,(D/L)$?

$Pr = \frac{C_p \mu}{k} = \frac{4180(0.001)}{0.6} = 6.97$

$Re\,Pr\,(D/L) = (13.3)(6.97)(0.16/2) = 0.74$

Use Fig 14.2-1 (BSLK); "const wall T (tube)"

$\frac{1}{0.74} = 1.35$, off chart to right. $Nu = 3.657$

$Nu = \frac{hD}{k} \rightarrow 3.657 = \frac{h(0.0016)}{0.6} \rightarrow h = 1371$

b) overall heat-transfer coef. U_0 (

$$\frac{1}{r_0 U_0} = \left(\frac{1}{r_0 h_0} + \frac{\ln(0.02/0.0008)}{17}\right) = (0.912 + 0.19) = \frac{1}{(0.0008)\,U_0}$$

$$\left[\frac{1}{(0.0008)(1371)} = 0.912\right] \qquad \hookrightarrow U_0 = 1134 \text{ W/m}^2\text{K}$$

(Note the thick ring of steel offers a relatively small part of the resistance to heat transfer. Most of the resistance is in heat transfer to the fluid.)

c) $\frac{U_0 D}{k} = \ln\left(\frac{T_0 - T_{b1}}{T_0 - T_{b2}}\right) Re\,Pr\,(D/(4L))$

$$\frac{(1134)(0.0008)}{0.6} = \ln\left(\frac{90-20}{90-T_{b2}}\right)(13.3)(6.97)(0.0008/(4 \times .02))$$

$$1.63 = \ln\left(\frac{90-20}{90-T_{b2}}\right) \rightarrow 5.11 = \frac{90-20}{90-T_{b2}} \rightarrow 76.3\,^\circ C$$

2. a) Since solid is at uniform T, can treat with macro balance.

heat in = accum [no convection in this part]

$$(2\pi R L) U_0 (T_s - T) = (\pi R^2 L)(\rho C_p) \frac{dT}{dt} \qquad T_s = T_{surroundings}$$

$$\frac{dT}{dt} \frac{1}{(T_s - T)} = -\frac{d(T_s - T)}{(T_s - T) dt} = -\frac{d \ln (T_s - T)}{dt} = \frac{2\pi R L U_0}{\pi R^2 L \rho C_p} = \frac{2 U_0}{R \rho C_p}$$

$$\ln (T_s - T) = \frac{-2 U_0}{R \rho C_p} t + C_1$$

$C_1 = \ln (T_s - T)$ at $t = 0$

$$\ln \left[\frac{T_s - T}{(T_s - T)_{t=0}} \right] = \frac{-2 U_0}{R \rho C_p} t = \frac{-2(14)}{(0.02)(2270)(999)} t = 0.000617 t$$

$$\ln \left[\frac{90 - 80}{90 - 20} \right] = -0.000617 t \rightarrow t = 3150 \text{ s} \quad [53 \text{ min}]$$

b) $\alpha = \frac{k}{\rho C_p} = \frac{2.61}{2270 (999)} = 1.15 \cdot 10^{-6}$

use Fig 11.5-2 for cylinder

we want $\frac{T - 20}{90 - 20} = \frac{T - T_0}{T_1 - T_0} = \frac{80 - 20}{90 - 20} = 0.86$ at $(r/R) = 0$

$\frac{\alpha t}{R^2} \approx 0.45$ [hard to see, since it's above chart]

$$0.45 = \frac{1.15 \cdot 10^{-6}}{(0.02)^2} t \rightarrow t = 157 \text{ s} \quad (2 \text{ min}, 37 \text{ sec})$$

c) These are resistances in series. Slowest estimate is best.

(a). Ans 71

3. First change sets dim'less T. $\frac{T-T_0}{T_1-T_0} = \frac{T-20}{120-20}$

The first change happened 150 s ago.

$$\frac{\alpha t}{R^2} = \frac{(1.15\cdot 10^{-6})(120)}{(0.02)^2} = 0.345 \qquad \Theta \cong 0.78$$

second change was down 30°C, or (−0.3) on this T scale.

After 60 s., $\frac{\alpha t}{R^2} = 0.1725 \qquad \Theta \approx 0.44$

$$\frac{T-20}{120-20} = 0.78 - (0.3)(0.44) = 0.648 \rightarrow T = 84.8$$

see note at end.

4. Shell energy balance. Shell of thickness Δx

conduction in $(LW)\,q_x|_x$

out $(LW)\,q_x|_{x+\Delta x}$

generation $(LW\Delta x)S$

no convection, accumulation

$(LW)q_x|_x - (LW)q_x|_{x+\Delta x} + (LW\Delta x)S = 0$ divide by $LW\Delta x$

$\frac{q_x|_x - q_x|_{x+\Delta x}}{\Delta x} + S = 0$ let $\Delta x \to 0$

$-\frac{dq_x}{dx} + S = 0 \rightarrow \frac{dq_x}{dx} = S$ integrate

$q_x = Sx + C_1$ B.C. 1 at $x=0$, $q_x = q^*$

$q_x = Sx + q^*$

b) Fourier's law. $-k\frac{dT}{dx} = q_x = Sx + q^*$

$\frac{dT}{dx} = -\frac{Sx}{k} - \frac{q^*}{k}$ Integrate

$T = -\frac{S}{k}\frac{x^2}{2} - \left(\frac{q^*}{k}\right)x + C_2$ B.C.: at $x=\delta$, $T=T_0$

$T_0 = -\frac{S}{k}\frac{\delta^2}{2} - \frac{q^*}{k}\delta + C_2$

$C_2 = \frac{S}{k}\frac{\delta^2}{2} + \frac{q^*}{k}\delta + T_0$

$\rightarrow T = \frac{S}{k}\frac{\delta^2}{2}\left(1-\left(\frac{x}{\delta}\right)^2\right) + \frac{q^*}{k}\delta\left(1-\frac{x}{\delta}\right) + T_0$

c) This problem is analogous to a falling film with a shear stress applied at the top surface. The solid is heated internally and cooled at the top surface. As q^* increases, the last location to cool is the bottom surface, at $x = \delta$. The minimum (negative) q^* is that where

$$\frac{dT}{dx} = 0 \text{ at } x = \delta$$

$$\frac{dT}{dx} = -\frac{Sx}{k} - \frac{q^*}{k} \quad \text{(see start of part (b))}.$$

$$\text{at } x = \delta \quad \frac{dT}{dx} = 0 = \frac{-S\delta}{k} - \frac{q^*}{k} \rightarrow q^* = S\delta$$

Note that $(S\delta)$ represents the heat released throughout the layer. If the heat flux at the top removes all the heat released throughout the layer, then the the whole layer is cooler than the bottom layer.

NOTE
Many students tried to use solutions in the book that don't apply here; e.g.

BSLK section 10.3 - but there is generation in the energy balance here

10.5: Newton's law of cooling is not involved here, and certainly not in the energy balance

10.6: the geometry here is rectangular, not cylindrical, here.

10.8: this is probably the closest analogue, but the generation term here is S, not related to fluid flow or a velocity gradient.

If a case in the book differs in the energy balance (geometry or terms in the balance) one has to start from the beginning. There might not be an example in the book that corresponds to some new problem.